MDE/M-24 **4666**

REAL ANALYSIS–II

Paper–MM–408

Time allowed : 3 Hours] [Maximum Marks : 80

Note : Attempt five questions in all, selecting one question from each unit. Question No. 9 is compulsory. All questions carry equal marks.

UNIT–I

1. (a) Let $\{E_n\}$ be infinite sequence of measurable sets such that $E_{n+1} \subset E_n$ for each n. Let $mE_1 < \infty$ then prove that: 8

$$m\left(\bigcap_{i=1}^{\infty} E_n\right) = \lim_{n \to \infty} mE_n.$$

(b) Show that Borel set is measurable. In particular each open set and each closed set is measurable. 8

2. (a) Prove that a constant function f defined on a measurable set E is measurable. 8

(b) Show that characteristics function χ_A is measurable if and only if A is measurable. 8

UNIT–II

3. (a) State and prove F. Riesz theorem. 8

(b) State and prove Lusin's Theorem. 8

P.T.O.

4. (a) If f is a bounded and measurable functions defined on a set of finite measure. If A and B are disjoint measurable sets then: 8

$$\int_{A \cup B} f = \int_A f + \int_B f$$

(b) Show that Bounded convergence theorem need not be true in case of Riemann integral. 8

UNIT–III

5. (a) Show and prove Fatou's lemma. 8

(b) Evaluate the Lebesgue integral of the function $f : [0, 1] \to R$ as: 8

$$f(x) = \begin{cases} x^{\frac{1}{1/3}} & \text{if } 0 < x \le 1 \\ 0 & \text{if } x = 0 \end{cases}$$

6. (a) State and prove Vitali's covering lemma. 16

UNIT–IV

7. (a) If f is bounded and measureable on [a, b] and

$$F(x) = \int_F^x f(t)\, dt + F(a)$$

then prove that $F'(x) = f(x)$ for almost all x in [a, b]. 8

(b) If f is absolutely continuous on [a, b] then show that it is bounded variation on [a, b]. 8

8. (a) Show that every convex function on an open internal is continuous. 6

(b) State and prove Riesz Representation theorem. 10

Compulsory Question

9. Attempt all questions: 8×2=16

(i) Prove that if f and g are measurable functions, then the set $\{x\ E : f(x) - g(x)\}$ is a measurable set.

(ii) State Egoroff's theorem.

(iii) Give an example of a function which is Riemann integrable but not Lebesgue integrable.

(iv) Define simple function.

(v) Define general Lebesgre integral.

(vi) State monotone convergence theorem.

(vii) Define absolutely continuous function.

(viii) State Minkowski inequalities of Lp spaces.